4-6 The Quadratic Formula and the Discriminant

Complete parts *a*–*c* for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

21. $2x^2 + 3x - 3 = 0$

SOLUTION:

a. Identify *a*, *b*, and *c* from the equation.

$a = 2$, $b = 3$ and $c = -3$.

Substitute the values in $b^2 - 4ac$ and simplify.

$$3^2 - 4(2)(-3) = 9 + 24$$
$$= 33$$

b. The discriminant is not a perfect square, so there are two irrational roots
.

c. Substitute the values of *a*, *b*, and *c* into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-3)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{33}}{4}$$

22. $4x^2 - 6x + 2 = 0$

SOLUTION:

a. Identify *a*, *b*, and *c* from the equation.

$a = 4$, $b = -6$ and $c = 2$.

Substitute the values in $b^2 - 4ac$ and simplify.

$$(-6)^2 - 4(4)(2) = 36 - 32$$
$$= 4$$

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of *a*, *b*, and *c* into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(2)}}{2(4)}$$
$$= \frac{6 \pm \sqrt{4}}{8}$$
$$= \frac{6 \pm 2}{8}$$
$$= \frac{1}{2} \text{ or } 1$$

23. $6x^2 + 5x - 1 = 0$

SOLUTION:

a. Identify *a*, *b*, and *c* from the equation.

$a = 6$, $b = 5$ and $c = -1$.

Substitute the values in $b^2 - 4ac$ and simplify.

$$5^2 - 4(6)(-1) = 25 + 24$$
$$= 49$$

b. The discriminant is a perfect square, so there are two rational roots
.

c. Substitute the values of *a*, *b*, and *c* into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(6)(-1)}}{2(6)}$$
$$= \frac{-5 \pm \sqrt{49}}{12}$$
$$= \frac{-5 \pm 7}{12}$$
$$= \frac{1}{6} \text{ or } -1$$

24. $6x^2 - x - 5 = 0$

SOLUTION:

a. Identify *a*, *b*, and *c* from the equation.

$a = 6$, $b = -1$ and $c = -5$.

Substitute the values in $b^2 - 4ac$ and simplify
.

$$(-1)^2 - 4(6)(-5) = 121$$

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of *a*, *b*, and *c* into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(6)(-5)}}{2(6)}$$
$$= \frac{1 \pm \sqrt{121}}{12}$$
$$= \frac{1 \pm 11}{12}$$
$$= 1 \text{ or } -\frac{5}{6}$$

25. $3x^2 - 3x + 8 = 0$

SOLUTION:

a. Identify *a*, *b*, and *c* from the equation.

$a = 3$, $b = -3$ and $c = 8$.

Substitute the values in $b^2 - 4ac$ and simplify

.

$$(-3)^2 - 4(3)(8) = -87$$

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of *a*, *b*, and *c* into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(8)}}{2(3)}$$

$$= \frac{3 \pm i\sqrt{87}}{6}$$

26. $2x^2 + 4x + 7 = 0$

SOLUTION:

a. Identify *a*, *b*, and *c* from the equation.

$a = 2$, $b = 4$ and $c = 7$.

Substitute the values in $b^2 - 4ac$ and simplify.

$$4^2 - 4(2)(7) = -40$$

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of *a*, *b*, and *c* into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(7)}}{2(2)}$$

$$= \frac{-4 \pm \sqrt{-40}}{4}$$

$$= \frac{-2 \pm i\sqrt{10}}{2}$$

27. $-5x^2 + 4x + 1 = 0$

SOLUTION:

a. Identify a, b, and c from the equation.

$a = -5$, $b = 4$ and $c = 1$.

Substitute the values in $b^2 - 4ac$ and simplify.

$$4^2 - 4(-5)(1) = 36$$

b. The discriminant is a perfect square, so there are two rational roots.

c. Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-5)(1)}}{2(-5)}$$
$$= \frac{-4 \pm \sqrt{36}}{-10}$$
$$= \frac{-4 \pm 6}{-10}$$
$$= 1 \text{ or } -\frac{1}{5}$$

28. $x^2 - 6x = -9$

SOLUTION:

a. Write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$x^2 - 6x = -9 \rightarrow 1x^2 - 6x + 9 = 0$$
$$a = 1$$
$$b = -6$$
$$c = 9$$

Substitute the values in $b^2 - 4ac$ and simplify.

$$(-6)^2 - 4(1)(9) = 0$$

b. The discriminant is 0, so there is one rational root.

c. Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{0}}{2}$$
$$= 3$$

29. $-3x^2 - 7x + 2 = 6$

SOLUTION:

a. Write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$-3x^2 - 7x + 2 = 6 \rightarrow -3x^2 - 7x - 4 = 0$$
$$a = -3$$
$$b = -7$$
$$c = -4$$

Substitute the values in $b^2 - 4ac$ and simplify.

$$(-7)^2 - 4(-3)(-4) = 1$$

b. The discriminant is 1, so there are two rational roots.

c. Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(-3)(-4)}}{2(-3)}$$
$$= \frac{7 \pm \sqrt{1}}{-6}$$
$$= \frac{7 \pm 1}{-6}$$
$$= -1 \text{ or } -\frac{4}{3}$$

30. $-8x^2 + 5 = -4x$

SOLUTION:

a. Write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$-8x^2 + 5 = -4x \rightarrow -8x^2 + 4x + 5 = 0$$
$$a = -8$$
$$b = 4$$
$$c = 5$$

Substitute the values in $b^2 - 4ac$ and simplify.

$$4^2 - 4(-8)(5) = 176$$

b. The discriminant is not a perfect square, so there are two irrational roots

.

c. Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-8)(5)}}{2(-8)}$$
$$= \frac{-4 \pm \sqrt{176}}{-16}$$
$$= \frac{-4 \pm 4\sqrt{11}}{-16}$$
$$= \frac{1 \pm \sqrt{11}}{4}$$

31. $x^2 + 2x - 4 = -9$

SOLUTION:

a. Write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$x^2 + 2x - 4 = -9 \rightarrow 1x^2 + 2x + 5 = 0$$
$$a = 1$$
$$b = 2$$
$$c = 5$$

Substitute the values in $b^2 - 4ac$ and simplify.

$$2^2 - 4(1)(5) = -16$$

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{-16}}{2}$$
$$= \frac{-2 \pm 4i}{2}$$
$$= -1 \pm 2i$$

32. $-6x^2 + 5 = -4x + 8$

SOLUTION:

a. Write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$-6x^2 + 5 = -4x + 8 \rightarrow -6x^2 + 4x - 3 = 0$$
$$a = -6$$
$$b = 4$$
$$c = -3$$

Substitute the values in $b^2 - 4ac$ and simplify.

$$4^2 - 4(-6)(-3) = -56$$

b. The discriminant is negative, so there are two complex roots.

c. Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(-6)(-3)}}{2(-6)}$$
$$= \frac{-4 \pm \sqrt{-56}}{-12}$$
$$= \frac{-4 \pm 2i\sqrt{14}}{-12}$$
$$= \frac{2 \pm i\sqrt{14}}{6}$$

33. **VIDEO GAMES** While Darnell is grounded his friend Jack brings him a video game. Darnell stands at his bedroom window, and Jack stands directly below the window. If Jack tosses a game cartridge to Darnell with an initial velocity of 35 feet per second, an equation for the height h feet of the cartridge after t seconds is $h = -16t^2 + 35t + 5$.

a. If the window is 25 feet above the ground, will Darnell have 0, 1, or 2 chances to catch the video game cartridge?

b. If Darnell is unable to catch the video game cartridge, when will it hit the ground?

SOLUTION:

a. Substitute 25 for y and simplify.

$$-16t^2 + 35t + 5 = 25$$
$$-16t^2 + 35t - 20 = 0$$

$a = -16$, $b = 35$ and $c = -20$.

Substitute the values in $b^2 - 4ac$ and simplify.

$$b^2 - 4ac = 35^2 - 4(-16)(-20)$$
$$= -55$$

Since the discriminant is negative, it has 0 real roots.

So, Darnell will have 0 chances to catch the video game cartridge.

b. Substitute 0 for h in the equation $h = -16t^2 + 35t + 5$

.

$$-16t^2 + 35t + 5 = 0$$

Identify a, b, and c from the equation.

$a = -16$, $b = 35$ and $c = 5$.

Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-35 \pm \sqrt{35^2 - 4(-16)(5)}}{2(-16)}$$
$$= \frac{-35 \pm \sqrt{1344}}{-32}$$
$$\approx 2.3 \text{ or } -0.05$$

As the time cannot be in negative it is about 2.3 seconds.

34. **CCSS SENSE-MAKING** Civil engineers are designing a section of road that is going to dip below sea level. The road's curve can be modeled by the equation $y = 0.00005x^2 - 0.06x$, where x is the horizontal distance in feet between the points where the road is at sea level and y is the elevation. The engineers want to put stop signs at the locations where the elevation of the road is equal to sea level. At what horizontal distances will they place the stop signs?

SOLUTION:

Substitute 0 for y in the equation $y = 0.00005x^2 - 0.06x$.

$$0.00005x^2 - 0.06x = 0$$

Identify a, b, and c from the equation.

$a = 0.00005$, $b = 0.06$ and $c = 0$.

Substitute the values of a, b, and c into the Quadratic Formula and simplify.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-0.06 \pm \sqrt{(0.06)^2 - 4(0.00005)(0)}}{2(0.00005)}$$
$$= \frac{-0.06 \pm \sqrt{0.0036}}{0.0001}$$
$$= \frac{-0.06 \pm 0.06}{0.0001}$$
$$= 0 \text{ or } 1200$$

The engineers will place the stop signs at 0 ft and 1200 ft.

46. **OPEN ENDED** Sketch the corresponding graph and state the number and type of roots for each of the following.

a. $b^2 - 4ac = 0$

b. A quadratic function in which $f(x)$ never equals zero.

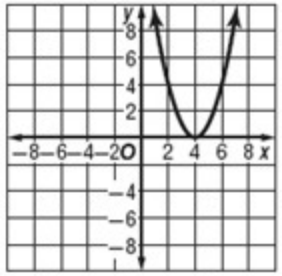
c. A quadratic function in which $f(a) = 0$ and $f(b) = 0$; $a \neq b$.

d. The discriminant is less than zero.

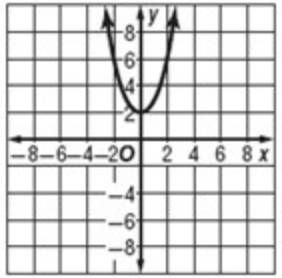
e. a and b are both solutions and can be represented as fractions.

SOLUTION:

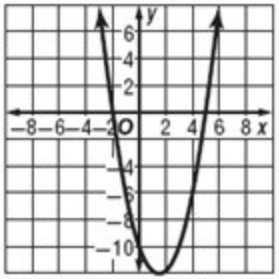
a. Sample answer: 1 rational root



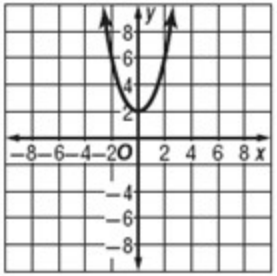
b. Sample answer: 2 complex roots



c. Sample answer: 2 real roots



d. Sample answer: 2 complex roots



e. Sample answer: 2 rational roots