4-3 Solving Quadratic Equations by Factoring

Write a quadratic equation in standard form with the given root(s).

17. 7

SOLUTION:

Write the pattern.

$$(x-p)(x-q)=0$$

Since there is only one root, it is a repeated root. Replace p and q with 7.

$$(x-7)(x-7)=0$$

Use the FOIL method to multiply.

$$\begin{aligned} x(x)+x(-7)-7(x)-7(-7)&=0\\ x^2-7x-7x+49&=0\\ x^2-14x+49&=0 \end{aligned}$$

19. $\frac{1}{5}, 6$

SOLUTION:
Write the pattern.

$$(x-p)(x-q)=0$$

Replace p and q with $\frac{1}{5}$ and 6.

$$\left(x-\frac{1}{5}\right)(x-6)=0$$

Use the FOIL method to multiply.

$$x(x)+x(-6)-\frac{1}{5}(x)-\frac{1}{5}(-6)=0$$
$$x^2-6x-\frac{1}{5}x+\frac{6}{5}=0$$

Multiply each side by 5.

$$5x^2-30x-x+6=0$$
$$5x^2-31x+6=0$$

Factor each polynomial.

21. $51c^3-34c$

SOLUTION:
The GCF of the two terms is $17c$. Factor the GCF.

$$51c^3-34c=17c\left(3c^2\right)-17c(2)$$
$$=17c\left(3c^2-2\right)$$

23. $3x^2 - 12$

SOLUTION:
Factor out 3.

$$3x^2 - 12 = 3\left(x^2 - 4\right)$$

Use the identity $a^2 - b^2 = (a + b)(a - b)$ to factor $x^2 - 4$.

$$x^2 - 4 = (x + 2)(x - 2)$$

Therefore,

$$3x^2 - 12 = 3(x + 2)(x - 2).$$

25. $48cg + 36cf - 4dg - 3df$

SOLUTION:
Factor $12c$ from the first two terms and $-d$ from the last two terms.

$$\begin{aligned} &48cg + 36cf - 4dg - 3df \\ &= 12c(4g + 3f) - d(4g + 3f) \end{aligned}$$

Factor $4g + 3f$ from the two terms.

$$\begin{aligned} &12c(4g + 3f) - d(4g + 3f) \\ &= (4g + 3f)(12c - d) \end{aligned}$$

Therefore,

$$\begin{aligned} &48cg + 36cf - 4dg - 3df \\ &= (4g + 3f)(12c - d) \end{aligned}$$.

27. $x^2 - 9x - 22$

SOLUTION:
Find the factors of –22 whose sum is –9.

2(–11) = –22 and 2 + (–11) = –9

Write –9*x* as 2*x* –11*x*.
$x^2 - 9x - 22 = x^2 + 2x - 11x - 22$

Factor *x* from the first two terms and –11 from the last two terms.
$x^2 + 2x - 11x - 22 = x(x+2) - 11(x+2)$

Factor *x* + 2 from the two terms.
$x(x+2) - 11(x+2) = (x+2)(x-11)$

Therefore,

$x^2 - 9x - 22 = (x+2)(x-11).$

29. $15x^2 + 7x - 2$

SOLUTION:
Here, *a* = 15, *b* = 7 and *c* = –2.

ac = 15(–2) = –30

Find two factors of –30 whose sum is 7.

10(–3) = –30 and 10 + (–3) = 7

Write 7*x* as 10*x* – 3*x*.

$15x^2 + 7x - 2 = 15x^2 + 10x - 3x - 2$

Factor 5*x* from the first two terms and –1 from the last two terms.
$15x^2 + 10x - 3x - 2 = 5x(3x+2) - 1(3x+2)$

Factor 3*x* + 2 from the two terms.

$5x(3x+2) - 1(3x+2) = (3x+2)(5x-1)$

Therefore,

$15x^2 + 7x - 2 = (3x+2)(5x-1).$

31. $18x^2 + 15x - 12$

SOLUTION:

Here, $a = 18$, $b = 15$ and $c = -12$.

$ac = 18(-12) = -216$

Find two factors of –216 whose sum is 15.

$24(-9) = -216$ and $24 + (-9) = 15$

Write $15x$ as $24x + (-9)x$.

$$18x^2 + 15x - 12 = 18x^2 + 24x - 9x - 12$$

Factor $6x$ from the first two terms and –3 from the last two terms.

$$18x^2 + 24x - 9x - 12 = 6x(3x + 4) - 3(3x + 4)$$

Factor $3x + 4$ from the two terms

.

$$\begin{aligned} 6x(3x+4) - 3(3x+4) &= (6x-3)(3x+4) \\ &= 3(2x-1)(3x+4) \end{aligned}$$

Therefore,

$$18x^2 + 15x - 12 = 3(2x-1)(3x+4).$$

33. $9x^2 - 25$

SOLUTION:

Use the identity $a^2 - b^2 = (a + b)(a - b)$

$$\begin{aligned} 9x^2 - 25 &= (3x)^2 - (5)^2 \\ &= (3x+5)(3x-5) \end{aligned}$$

Therefore,

$$9x^2 - 25 = (3x+5)(3x-5).$$

35. $15x^2 - 84x - 36$

SOLUTION:

Factor 3 from all the three terms.

$$15x^2 - 84x - 36 = 3\left(5x^2 - 28x - 12\right)$$

Factor $5x^2 - 28x - 12$.

Here, $a = 5$, $b = -28$ and $c = -12$.

$ac = 5(-12) = -60$

Find two factors of -60 whose sum is -28.

$-30(2) = -60$ and $-30 + 2 = -28$

Write $-28x$ as $-30x + 2x$.

$$5x^2 - 28x - 12 = 5x^2 - 30x + 2x - 12$$

Factor $5x$ from the first two terms and 2 from the last two terms.

$$5x^2 - 30x + 2x - 12 = 5x\left(x - 6\right) + 2\left(x - 6\right)$$

Factor $x - 6$ from the two terms.

$$5x\left(x - 6\right) + 2\left(x - 6\right) = \left(5x + 2\right)\left(x - 6\right)$$

Therefore,

$$15x^2 - 84x - 36 = 3\left(5x + 2\right)\left(x - 6\right).$$

37. $12xy^2 - 108x$

SOLUTION:

Factor out the GCF, $12x$.

$$12xy^2 - 108x = 12x\left(y^2 - 9\right)$$

Use the identity $a^2 - b^2 = (a + b)(a - b)$ to factor $y^2 - 9$.

$$y^2 - 9 = (y + 3)(y - 3)$$

Therefore,

$$12xy^2 - 108x = 12x(y + 3)(y - 3).$$

4-3 Solving Quadratic Equations by Factoring

Solve each equation by factoring.

39. $x^2 - 5x - 24 = 0$

SOLUTION:

Find the factors of –24 whose sum is –5.

3(–8) = –24 and 3 + (–8) = –5

Write $-5x$ as $3x + (-5x)$.

$$x^2 - 5x - 24 = 0$$
$$x^2 + 3x - 8x - 24 = 0$$

Factor x from the first two terms and –8 from the last two terms.

$$x^2 + 3x - 8x - 24 = 0$$
$$x(x+3) - 8(x+3) = 0$$

Factor $x + 3$ from the two terms.

$$x(x+3) - 8(x+3) = 0$$
$$(x+3)(x-8) = 0$$

Use the Zero Product Property.

$$(x+3)(x-8) = 0 \Rightarrow x+3 = 0 \quad \text{or} \quad x-8 = 0$$
$$\Rightarrow x = -3 \quad \text{or} \quad x = 8$$

Therefore, the roots are –3 and 8.

4-3 Solving Quadratic Equations by Factoring

41. $x^2 + 13 = 17$

SOLUTION:

Write the equation with right side equal to zero.

$$x^2 + 13 - 17 = 0$$
$$x^2 - 4 = 0$$

Use the identity $a^2 - b^2 = (a + b)(a - b)$ to factor $x^2 - 4$.

$$x^2 - 4 = (x + 2)(x - 2) = 0$$

Use the Zero Product Property.

$$(x + 2)(x - 2) = 0 \Rightarrow x + 2 = 0 \quad \text{or} \quad x - 2 = 0$$
$$\Rightarrow x = -2 \quad \text{or} \quad x = 2$$

Therefore, the roots are –2 and 2.

43. $-8x^2 + 46x - 30 = 0$

SOLUTION:
Factor out –1.

$$-1\left(8x^2 - 46x + 30\right) = 0$$
$$8x^2 - 46x + 30 = 0$$

Now factor $8x^2 - 46x + 30$.

Here, $a = 8$, $b = -46$ and $c = 30$.

$ac = 8(30) = 240$

Find two factors of 240 whose sum is –46.

$-40(-6) = 240$ and $-40 + (-6) = -46$

Write $-46x$ as $-40x + (-6x)$.

$$8x^2 - 46x + 30 = 8x^2 - 40x - 6x + 30$$

Factor $8x$ from the first two terms and –6 from the last two terms.

$$8x^2 - 40x - 6x + 30 = 0$$
$$8x(x-5) - 6(x-5) = 0$$

Factor $x - 5$ from the two terms.

$$(x-5)(8x-6) = 0$$

Use the Zero Product Property.

$$(x-5)(8x-6) = 0 \Rightarrow x - 5 = 0 \quad \text{or} \quad 8x - 6 = 0$$
$$\Rightarrow x = 5 \qquad \text{or} \quad x = \frac{6}{8}$$
$$= \frac{3}{4}$$

Therefore, the roots are 5 and $\frac{3}{4}$.