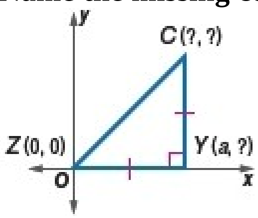
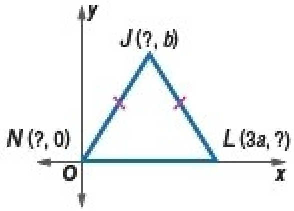
Name the missing coordinate(s) of each triangle.

13. 

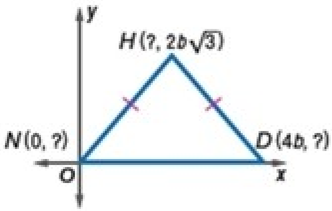
SOLUTION:

ΔZCY has two congruent sides, so it is isosceles. Here, $ZY = CY$. The point Y is on the x-axis, so its y-coordinate is 0. The coordinates of Y are $(a, 0)$. So, $ZY = CY = a$. The point C lies on the line $x = a$. So, its x-coordinate is a. It lies a units above the x-axis since $CY = a$. So, its y-coordinate is a. Hence C is (a, a).

15. 

SOLUTION:

ΔNJL has two congruent sides, so it is isosceles. Here, $NJ = JL$. Point N is at origin, so its coordinates are (0, 0). Point L is on the x-axis, so its y-coordinate is 0. The coordinates of L are $(3a, 0)$. Find the coordinates of point J, that makes the triangle isosceles. This point is half way between N and L, so the x-coordinate is $1.5a$. Let b be the y-coordinate of J. So, the coordinates of J should be $(1.5a, b)$.

17. 

SOLUTION:

ΔNHD has two congruent sides, so it is isosceles. Here, $NH = HD$. Point N is at origin, so its coordinates are (0, 0). Point D is on the x-axis, so its y-coordinate is 0. The coordinates of D are $(4b, 0)$. Find the x-coordinate of point H, that makes the triangle isosceles. This point is half way between N and D, so the x-coordinate of H should be $\frac{4b}{2}$ or $2b$. Therefore, the coordinates of point H are $\left(2b, 2b\sqrt{3}\right)$.

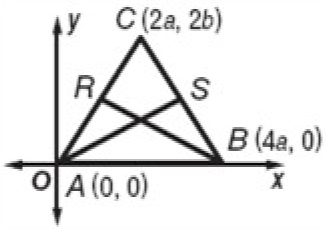
CCSS ARGUMENTS Write a coordinate proof for each statement.

19. The segments joining the base vertices to the midpoints of the legs of an isosceles triangle are congruent.

SOLUTION:

Draw isosceles triangle *ABC* on a coordinate plane, find the midpoints *R* and *S* of the two legs, and show that the segments connecting each midpoint with the opposite vertex have the same length. Start by placing a vertex at the origin and label it *A*. Use coordinates that are multiples of 2 because the Midpoint Formula involves dividing the sum of the coordinates by 2. So, place point *B* on the *x*-axis at $(4a, 0)$. Since the triangle is isosceles, the *x*-coordinate of point *C* is halfway between 0 and $4a$ or $2a$. We cannot write the *y*-coordinate in terms of *a*, so call it $2b$.

Given: Isosceles $\triangle ABC$ with $\overline{AC} \cong \overline{BC}$;
R and *S* are midpoints of legs $\overline{AC}$ and $\overline{BC}$.



Prove: $\overline{AS} \cong \overline{BR}$

Proof:

The coordinates of *S* are $\left(\frac{2a+4a}{2}, \frac{2b+0}{2}\right)$ or $(3a, b)$.

The coordinates of *R* are $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right)$ or (a, b).

$AS = \sqrt{(3a-0)^2 + (b-0)^2}$ or $\sqrt{9a^2 + b^2}$

$BR = \sqrt{(4a-a)^2 + (0-b)^2}$ or $\sqrt{9a^2 + b^2}$

Since $AS = BR$, $\overline{AS} \cong \overline{BR}$.

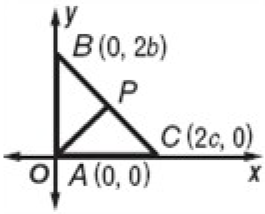
PROOF Write a coordinate proof for each statement.

21. The measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

SOLUTION:

Draw right triangle ABC on a coordinate plane, find the midpoint P of the hypotenuse, and show that the length of the segment joining the midpoint and the vertex of the right angle is one-half the length of the hypotenuse. Start by placing a vertex at the origin and label it A. Use coordinates that are multiples of 2 because the Midpoint Formula involves dividing the sum of the coordinates by 2. Place point C on the x-axis at $(2c, 0)$. Since the triangle is right, place point B on the y-axis at $(0, 2b)$.

Given: Right $\triangle ABC$ with right $\angle BAC$; P is the midpoint of $\overline{BC}$.



Prove: $AP = \frac{1}{2}BC$

Proof:

Midpoint P is $\left(\frac{0+2c}{2}, \frac{2b+0}{2}\right)$ or (c, b).

$AP = \sqrt{(c-0)^2 + (b-0)^2}$ or $\sqrt{c^2 + b^2}$

$BC = \sqrt{(2c-0)^2 + (0-2b)^2} = \sqrt{4c^2 + 4b^2}$ or $2\sqrt{c^2 + b^2}$

$\frac{1}{2}BC = \sqrt{c^2 + b^2}$

So, $AP = \frac{1}{2}BC.$