

1.4 -- Absolute Value Equations

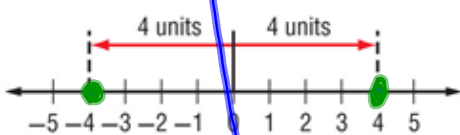
Must always be positive

Key Concept Absolute Value refers to a point's distance from 0.

Words For any real number a , if a is positive or zero, the absolute value of a is a . If a is negative, the absolute value of a is the opposite of a .

Symbols For any real number a , $|a| = a$ if $a \geq 0$, and $|a| = -a$ if $a < 0$.

Model $|-4| = 4$ and $|4| = 4$



A number line from -5 to 5 with tick marks at every integer. Green dots are placed at -4 and 4. Red double-headed arrows indicate the distance from 0 to -4 and from 0 to 4, both labeled "4 units".

abs. value of negative four is four.

opposite

Ex.1: Evaluate $2.7 + |6 - 2x|$ if $x = 4$.

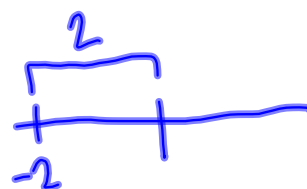
$$2.7 + |6 - 2(4)|$$

$$2.7 + |6 - 8|$$

$$2.7 + |-2|$$

$$2.7 + 2 \rightarrow$$

$$\boxed{4.7}$$



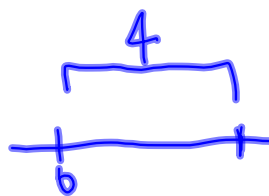
Ex.2: What is the value of $2.3 - |3y - 10|$ if $y = 2$?

$$2.3 - |3(2) - 10|$$

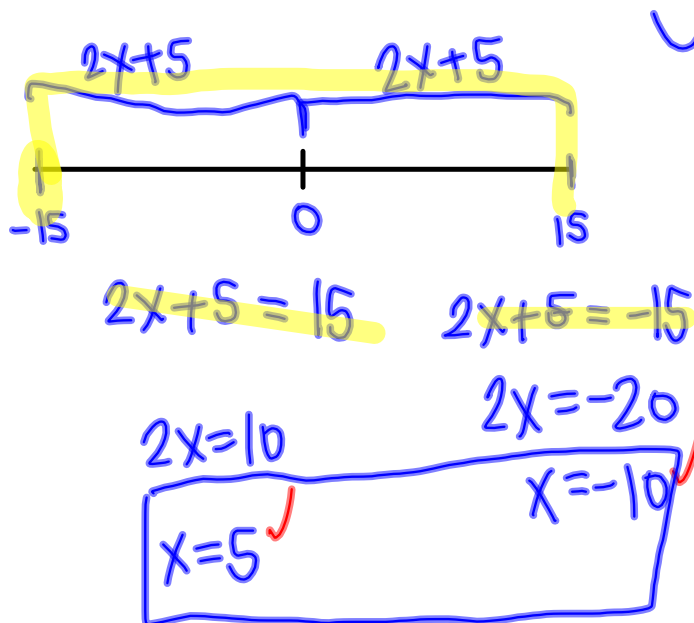
$$2.3 - |-4|$$

$$2.3 - 4$$

$$\boxed{-1.7}$$



Ex.3: What is the solution to $|2x + 5| = 15$?



a
 Check solutions by plugging into orig.
 $|2(5)+5| \stackrel{?}{=} 15$
 $|15| = 15$ ✓
 $|2(-10)+5| \stackrel{?}{=} 15 \rightarrow |-15|$

Ex. 4: Solve $|6 - 4t| + 5 = 0$.

* Abs. value means distance!

$|6 - 4t| = -5$ **can't be negative!*

No solution

Common error

$6 - 4t = -5$

$-4t = -11$

$t = +\frac{11}{4}$

$6 - 4t = 5$

$-4t = -1$

$t = \frac{1}{4}$

 $|6 - 4(\frac{11}{4})| + 5 = 0$
 $|6 - 11| + 5 = 0$
 $|5| + 5$
 $5 + 5 \neq 0$

} $|6 - 4(\frac{1}{4})| + 5 = 0$
 $|6 - 1| + 5 = 0$
 $|5| + 5$
 $5 + 5 \neq 0$

Ex.5: $\frac{7}{7}|-4+x| = \frac{63}{7}$ * do not distribute
 ** isolate abs. value

$|-4+x| = 9$

$-4+x=9$
 $x=13$

$-4+x=-9$
 $x=-5$

$7|-4+13| \stackrel{?}{=} 63$
 $7|9| = 63$
 $7 \cdot 9 = 63 \checkmark$

$7|-4+(-5)| \stackrel{?}{=} 63$
 $7|-9|$
 $7 \cdot 9 = 63 \checkmark$

$$\text{Ex.6: } -6|4x| = -24$$

$$|4x| = 4$$

$$4x = 4$$

$$x = 1$$

$$4x = -4$$

$$x = -1$$

$$\text{Ex.7: } -3|m-3| + 9 = 3$$

$$\frac{-3}{-3} |m-3| = \frac{-9}{-3}$$

$$|m-3| = 2$$

$$m-3=2$$

$$\boxed{m=5}$$

$$m-3=-2$$

$$\boxed{m=1}$$

Ex.8: $10 \mid 8 - 3n \mid - 7 = 93$
let $a = |8 - 3n|$

$$10 \mid 8 - 3n \mid = 100$$

$$|8 - 3n| = 10$$

$$8 - 3n = 10$$

$$8 - 3n = -10$$

$$10a - 7 = 93$$

$$a = 10$$

Ex.9: Martin makes exercise weights. For his 10-pound dumbbells, he guarantees that the actual weight of his dumbbells is within 0.1 pound of 10 pounds. Write and solve an equation that describes the minimum and maximum weight of his 10-pound dumbbells.

central value

range

$$|x - c| = r$$
$$|W - 10| = 0.1$$
$$\begin{array}{l} W - 10 = 0.1 \\ +10 \quad +10 \\ \hline W = 10.1 \end{array}$$
$$\begin{array}{l} W - 10 = -0.1 \\ +10 \quad +10 \\ \hline W = 9.9 \end{array}$$