1-4 Solving Absolute Value Equations

22. **FOOD** To make cocoa powder, cocoa beans are roasted. The ideal temperature for roasting is 300°F, plus or minus 25°. Write and solve an equation describing the maximum and minimum roasting temperatures for cocoa beans.

SOLUTION:

$|x - 300| = 25$

Solve the equation $|x - 300| = 25$.

Case 1:	Case 2:
$x - 300 = 25$	$x - 300 = -25$
$x - 300 + 300 = 25 + 300$	$x - 300 + 300 = -25 + 300$
$x = 325$	$x = 275$

So, the maximum temperature is 325°F and the minimum temperature is 275°F.

Solve each equation. Check your solutions.

23. $|z - 13| = 21$

SOLUTION:

Case 1:	Case 2:
$z - 13 = 21$	$z - 13 = -21$
$z - 13 + 13 = 21 + 13$	$z - 13 + 13 = -21 + 13$
$z = 34$	$z = -8$

There appear to be two solutions, 34 and –8.
Check: Substitute each value in the original equation.

$\|z - 13\| = 21$	$\|z - 13\| = 21$
$\|34 - 13\| \stackrel{?}{=} 21$	$\|-8 - 13\| \stackrel{?}{=} 21$
$\|21\| \stackrel{?}{=} 21$	$\|-21\| \stackrel{?}{=} 21$
$21 = 21$✓	$21 = 21$✓

The solution set is $\{34, -8\}$.

25. $9 = |d + 5|$

SOLUTION:

Case 1:	Case 2:
$d + 5 = 9$	$d + 5 = -9$
$d + 5 - 5 = 9 - 5$	$d + 5 - 5 = -9 - 5$
$d = 4$	$d = -14$

There appear to be two solutions, 4 and –14.
Check: Substitute each value in the original equation.

$9 = \|d + 5\|$	$9 = \|d + 5\|$
$9 \stackrel{?}{=} \|4 + 5\|$	$9 \stackrel{?}{=} \|-14 + 5\|$
$9 \stackrel{?}{=} \|9\|$	$9 \stackrel{?}{=} \|-9\|$
$9 = 9$✓	$9 = 9$✓

The solution set is $\{4, -14\}$.

27. $5|q + 6| = 20$

SOLUTION:

$$5|q + 6| = 20$$
$$\frac{5|q + 6|}{5} = \frac{20}{5}$$
$$|q + 6| = 4$$

Case 1:	Case 2:
$q + 6 = 4$	$q + 6 = -4$
$q + 6 - 6 = 4 - 6$	$q + 6 - 6 = -4 - 6$
$q = -2$	$q = -10$

There appear to be two solutions, –2 and –10.
Check: Substitute each value in the original equation.

$5\|q + 6\| = 20$	$5\|q + 6\| = 20$
$5\|-2 + 6\| \stackrel{?}{=} 20$	$5\|-10 + 6\| \stackrel{?}{=} 20$
$5\|4\| \stackrel{?}{=} 20$	$5\|-4\| \stackrel{?}{=} 20$
$5(4) \stackrel{?}{=} 20$	$5(4) \stackrel{?}{=} 20$
$20 = 20$✓	$20 = 20$✓

The solution set is $\{-2, -10\}$.

29. $3|2a-4|=0$

SOLUTION:

$$\begin{aligned}
3|2a-4| &= 0\\
\frac{3|2a-4|}{3} &= \frac{0}{3}\\
|2a-4| &= 0\\
2a-4 &= 0\\
2a-4+4 &= 0+4\\
2a &= 4\\
\frac{2a}{2} &= \frac{4}{2}\\
a &= 2
\end{aligned}$$

Check: Substitute $a = 2$ in the original equation.

$$\begin{aligned}
3|2a-4| &= 0\\
3|2(2)-4| &\stackrel{?}{=} 0\\
3|4-4| &\stackrel{?}{=} 0\\
3|0| &\stackrel{?}{=} 0\\
3(0) &\stackrel{?}{=} 0\\
0 &= 0 \checkmark
\end{aligned}$$

The solution is $a = 2$.

31. $2|3x-4|+8=6$

SOLUTION:

$$\begin{aligned}
2|3x-4|+8 &= 6\\
2|3x-4|+8-8 &= 6-8\\
2|3x-4| &= -2\\
\frac{2|3x-4|}{2} &= \frac{-2}{2}\\
|3x-4| &= -1
\end{aligned}$$

Case 1:

$$\begin{aligned}
3x-4 &= -1\\
3x-4+4 &= -1+4\\
3x &= 3\\
\frac{3x}{3} &= \frac{3}{3}\\
x &= 1
\end{aligned}$$

Case 2:

$$\begin{aligned}
3x-4 &= -(-1)\\
3x-4 &= 1\\
3x-4+4 &= 1+4\\
3x &= 5\\
\frac{3x}{3} &= \frac{5}{3}\\
x &= \frac{5}{3}
\end{aligned}$$

There appear to be two solutions, 1 and $\frac{5}{3}$.

Check: Substitute the values in the original equation.

$$\begin{aligned}
2|3x-4|+8 &= 6\\
2|3(1)-4|+8 &\stackrel{?}{=} 6\\
2|3-4|+8 &\stackrel{?}{=} 6\\
2|-1|+8 &\stackrel{?}{=} 6\\
2(1)+8 &\stackrel{?}{=} 6\\
2+8 &\stackrel{?}{=} 6\\
10 &\neq 6
\end{aligned}$$

$$\begin{aligned}
2|3x-4|+8 &= 6\\
2\left|3\left(\frac{5}{3}\right)-4\right|+8 &\stackrel{?}{=} 6\\
2|5-4|+8 &\stackrel{?}{=} 6\\
2|1|+8 &\stackrel{?}{=} 6\\
2(1)+8 &\stackrel{?}{=} 6\\
2+8 &\stackrel{?}{=} 6\\
10 &\neq 6
\end{aligned}$$

Since $10 \neq 6$, the solution set is $\varnothing$.

33. $-3|3t-2|-12=-6$

SOLUTION:

$$\begin{aligned}-3|3t-2|-12&=-6\\-3|3t-2|-12+12&=-6+12\\-3|3t-2|&=6\\\frac{-3|3t-2|}{-3}&=\frac{6}{-3}\\|3t-2|&=-2\end{aligned}$$

Case 1:

$$\begin{aligned}3t-2&=-2\\3t-2+2&=-2+2\\3t&=0\\\frac{3t}{3}&=\frac{0}{3}\\t&=0\end{aligned}$$

Case 2:

$$\begin{aligned}3t-2&=-(-2)\\3t-2&=2\\3t-2+2&=2+2\\3t&=4\\\frac{3t}{3}&=\frac{4}{3}\\t&=\frac{4}{3}\end{aligned}$$

There appear to be two solutions, 0 and $\frac{4}{3}$.

Check: Substitute the values in the original equation.

$$\begin{aligned}-3|3t-2|-12&=-6\\-3|3(0)-2|-12&\stackrel{?}{=}-6\\-3|0-2|-12&\stackrel{?}{=}-6\\-3|-2|-12&\stackrel{?}{=}-6\\-3|-2|-12+12&\stackrel{?}{=}-6+12\\-3|-2|&\stackrel{?}{=}6\\-3(2)&\stackrel{?}{=}6\\-6&\neq 6\end{aligned}$$

$$\begin{aligned}-3|3t-2|-12&=-6\\-3\left|3\left(\frac{4}{3}\right)-2\right|-12&\stackrel{?}{=}-6\\-3|4-2|-12&\stackrel{?}{=}-6\\-3|2|-12&\stackrel{?}{=}-6\\-3|2|-12+12&\stackrel{?}{=}-6+12\\-3|2|&\stackrel{?}{=}6\\-3(2)&\stackrel{?}{=}6\\-6&\neq 6\end{aligned}$$

Since $-6 \neq 6$, the solution set is $\varnothing$.

35. **MONEY** The U.S. Mint produces quarters that weigh about 5.67 grams each. After the quarters are produced, a machine weighs them. If the quarter weighs 0.02 gram more or less than the desired weight, the quarter is rejected. Write and solve an equation to find the heaviest and lightest quarters the machine will approve.

SOLUTION:

Substitute $c = 5.67$ and $r = 0.02$ in the equation $|x - c| = r$.

$|x - 5.67| = 0.02$

Solve the equation $|x - 5.67| = 0.02$.

Case 1:

$$\begin{aligned}x-5.67&=0.02\\x-5.67+5.67&=0.02+5.67\\x&=5.69\end{aligned}$$

Case 2:

$$\begin{aligned}x-5.67&=-0.02\\x-5.67+5.67&=-0.02+5.67\\x&=5.65\end{aligned}$$

So, the heaviest quarters the machine will approve are those weighing 5.69 grams. The lightest quarters the machine will approve is 5.65 grams.

Evaluate each expression if $q = -8$, $r = -6$, and $t = 3$.

37. $2q + |2rt + q|$

SOLUTION:

Substitute –8 for q, –6 for r, and 3 for t and solve.

$$\begin{aligned} 2q + |2rt + q| &= 2(-8) + |2(-6)(3) + (-8)| \\ &= -16 + |-36 - 8| \\ &= -16 + |-44| \\ &= -16 + 44 \\ &= 28 \end{aligned}$$