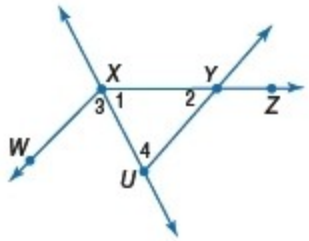
1-4 Angle Measure

Use the figure at the right.



2. Name the sides of $\angle 3$.

SOLUTION:

$\overrightarrow{XW}$, $\overrightarrow{XU}$

3. What is another name for $\angle 2$?

SOLUTION:

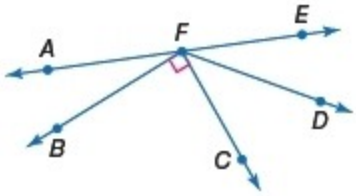
$\angle XYU$, $\angle UYX$

4. What is another name for $\angle UXY$?

SOLUTION:

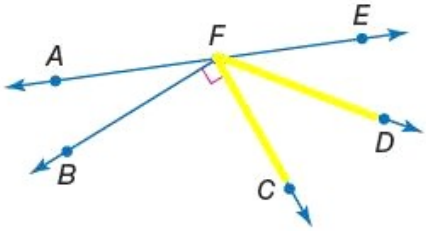
$\angle 1$, $\angle YXU$

Copy the diagram shown, and extend each ray. Classify each angle as *right*, *acute*, or *obtuse*. Then use a protractor to measure the angle to the nearest degree.



5. $\angle CFD$

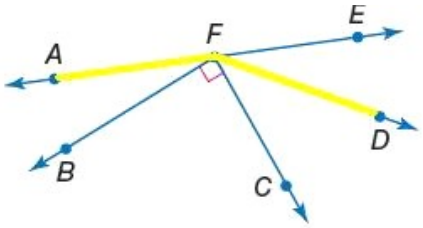
SOLUTION:



$\angle$CFD is an acute angle. The measure of $\angle$CFD is 40.

6. $\angle AFD$

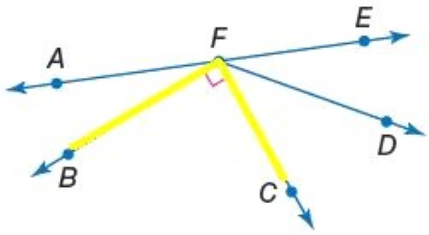
SOLUTION:



$\angle$ AFD is an obtuse angle. The measure of $\angle$AFD is 150.

7. $\angle BFC$

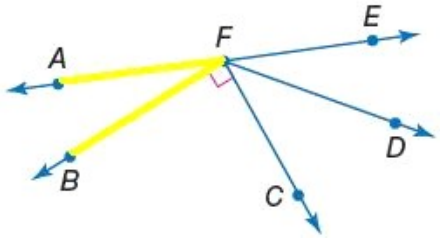
SOLUTION:



$\angle$BFC is a right angle. The measure of $\angle$BFC is 90.

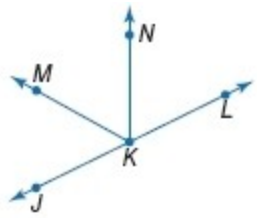
8. $\angle AFB$

SOLUTION:



$\angle$ AFB is an acute angle. The measure of $\angle$AFB is 25

ALGEBRA In the figure, $\overrightarrow{KJ}$ and $\overrightarrow{KL}$ are opposite rays, $\overrightarrow{KN}$ bisects $\angle LKM$.



9. If $m\angle LKM = 7x - 5$ and $m\angle NKM = 3x + 9$, find $m\angle LKM$.

SOLUTION:

$m\angle LKM = m\angle LKN + m\angle NKM$. Because of the definition of bisecting rays, $m\angle LKN = m\angle NKM$.

$m\angle LKM = m\angle LKN + m\angle NKM$	Definition of bisecting ray
$m\angle LKM = m\angle NKM + m\angle NKM$	Substitution
$m\angle LKM = 2(m\angle NKM)$	Addition.
$7x - 5 = 2(3x + 9)$	Substitution.
$7x - 5 = 6x + 18$	Distributive Property
$7x - 6x - 5 = 6x - 6x + 18$	Subtract 6x from each side.
$x - 5 = 18$	Simplify.
$x - 5 + 5 = 18 + 5$	Add 5 to each side.
$x = 23$	Simplify.

To find , $m\angle LKM$ substitute $x = 23$ into the equation $m\angle LKM = 7x - 5$,.

$$\begin{aligned} 7x - 5 &= 7(23) - 5 \\ &= 161 - 5 \\ &= 156 \end{aligned}$$

10. If $m\angle NKL = 7x - 9$ and $m\angle JKM = x + 3$, find $m\angle JKN$.

SOLUTION:

By the Angle Addition Postulate, $m\angle MKJ + m\angle NKM + m\angle LKN = 180$.

Since $\overrightarrow{KN}$ bisects $\angle LKM$, $m\angle NKM = m\angle NKL$.

Thus, $m\angle MKJ + m\angle LKN + m\angle LKN = 180$.

$m\angle MKJ + m\angle LKN + m\angle LKN = 180$	
$x + 3 + 7x - 9 + 7x - 9 = 180$	Substitution.
$15x - 15 = 180$	Simplify.
$15x - 15 + 15 = 180 + 15$	Add 15 to each side.
$15x = 195$	Simplify.
$\frac{15x}{15} = \frac{195}{15}$	Divide each side by 15.
$x = 13$	Simplify.

Find an expression for $m\angle JKN$.

$$\begin{aligned} m\angle JKN &= m\angle JKM + m\angle NKM \\ &= x + 3 + 7x - 9 \\ &= 8x - 6 \end{aligned}$$

To find $8x - 6$, substitute the value 13 for x.

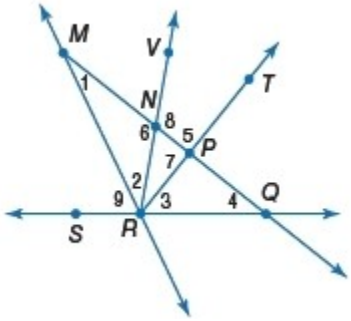
$$\begin{aligned} m\angle JKN &= 8(13) - 6 \\ &= 104 - 6 \\ &= 98 \end{aligned}$$

11. **CCSS PRECISION** A miter cut is used to build picture frames with corners that meet at right angles.
a. José miters the ends of some wood for a picture frame at congruent angles. What is the degree measure of his cut? Explain and classify the angle.
b. What does the joint represent in relation to the angle formed by the two pieces?

SOLUTION:

a. 45; When joined together, the angles form a right angle, which measures 90. If the two angles that form this right angle are congruent, then the measure of each angle is 90 ÷ 2 or 45. The angle of the cut is an acute angle.
b. The joint is the angle bisector of the frame angle.

For Exercises 12–29, use the figure below.



Name the vertex of each angle.

12. $\angle 4$

SOLUTION:
Q

13. $\angle 7$

SOLUTION:
P

14. $\angle 2$

SOLUTION:
R

15. $\angle 1$

SOLUTION:
M

Name the sides of each angle.

16. $\angle TPQ$

SOLUTION:
$\overrightarrow{PT}, \overrightarrow{PQ}$

17. $\angle VNM$

SOLUTION:
$\overrightarrow{NV}, \overline{NM}$

18. $\angle 6$

SOLUTION:
$\overline{NM}, \overline{NR}$

19. $\angle 3$

SOLUTION:
$\overrightarrow{RP}, \overrightarrow{RQ}$

Write another name for each angle.

20. $\angle 9$

SOLUTION:
$\angle MRS$, $\angle SRM$

21. $\angle QPT$

SOLUTION:
$\angle TPQ$

22. $\angle MQS$

SOLUTION:
$\angle 4$, $\angle SQM$, $\angle MQR$, $\angle RQM$, $\angle NQS$, $\angle SQN$, $\angle NQR$, $\angle RQN$, $\angle PQR$, $\angle RQP$, $\angle PQS$, $\angle SQP$

23. $\angle 5$

SOLUTION:
$\angle TPN$, $\angle NPT$, $\angle TPM$, $\angle MPT$

24. Name an angle with vertex *N* that appears obtuse.

SOLUTION:
Sample answer: $\angle VNQ$

25. Name an angle with vertex *Q* that appears acute.

SOLUTION:
$\angle 4$

26. Name a point in the interior of $\angle VRQ$.

SOLUTION:
P, *T*

27. Name a point in the exterior of $\angle MRT$.

SOLUTION:
S, *Q*

28. Name a pair of angles that share exactly one point.

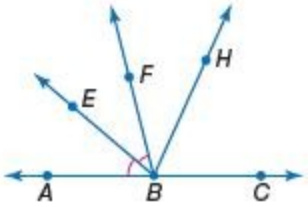
SOLUTION:
Sample answer: $\angle 6$ and $\angle 8$ share the vertex point *N*.

29. Name a pair of angles that share more than one point.

SOLUTION:

Sample answer: $\angle MPR$ and $\angle PRQ$ share points P and R.

ALGEBRA In the figure, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays. $\overrightarrow{BH}$ bisects $\angle EBC$.



37. If $m\angle ABE = 2n + 7$ and $m\angle EBF = 4n - 13$, find $m\angle ABE$.

SOLUTION:

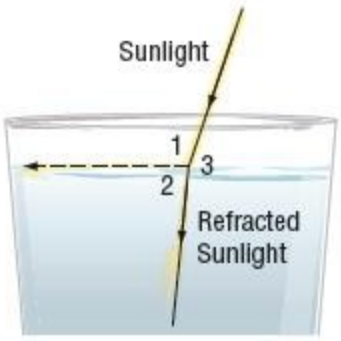
Since $\angle ABE \cong \angle EBF$, $m\angle ABE = m\angle EBF$.
So, $2n + 7 = 4n - 13$.

$m\angle ABE = m\angle EBF$	
$2n + 7 = 4n - 13$	Substitution.
$2n - 4n + 7 = 4n - 4n - 13$	Subtract $4n$ from each side.
$-2n + 7 = -13$	Simplify.
$-2n + 7 - 7 = -13 - 7$	Subtract 7 from each side.
$-2n = -20$	Simplify.
$\frac{-2n}{-2} = \frac{-20}{-2}$	Divide each side by -2.
$n = 10$	Simplify.

Substitute.

$$\begin{aligned} m\angle ABE &= 2n + 7 \\ &= 2(10) + 7 \\ &= 27 \end{aligned}$$

47. **PHYSICS** When you look at a pencil in water, it looks bent. This illusion is due to *refraction*, or the bending of light when it moves from one substance to the next.
Refer to Page 43.



a. What is $m\angle 1$? Classify this angle as *acute*, *right*, or *obtuse*.
b. What is $m\angle 2$? Classify this angle as *acute*, *right*, or *obtuse*.
c. Without measuring, determine how many degrees the path of the light changes after it enters the water. Explain your reasoning.

SOLUTION:

a. Use a protractor to measure $\angle 1$ in the diagram. The measure should be about 110; Since the measure is greater than 90 but less than 180, the angle is obtuse.
b. Use a protractor to measure $\angle 2$ in the diagram. The measure should be about 85; Since the measure is less than 90, the angle is acute.
c. about 15; If the original path of the light is extended, the measure of the angle the original path makes with the refracted path represents the number of degrees the path of the light changed. The sum of the measure of this angle and the $m\angle 3$ is 180. The measure of $\angle 3$ is 360 – (110 + 85) or 165, so the measure of the angle the original path makes with the refracted path is 180 – 165 or 15.